Exercise 60

At what point on the curve $y = 1 + 2e^x - 3x$ is the tangent line parallel to the line 3x - y = 5? Illustrate by graphing the curve and both lines.

Solution

Writing the given equation of the line as

$$y = 3x - 5,$$

we see that it has a slope of 3. The aim is to take the derivative of the given function and find where it's equal to 3.

$$y' = \frac{d}{dx}(1 + 2e^x - 3x)$$

= $\frac{d}{dx}(1) + \frac{d}{dx}(2e^x) - \frac{d}{dx}(3x)$
= $\frac{d}{dx}(1) + 2\frac{d}{dx}(e^x) - 3\frac{d}{dx}(x)$
= $(0) + 2(e^x) - 3(1)$
= $2e^x - 3$

Set this equal to 3 and solve for x.

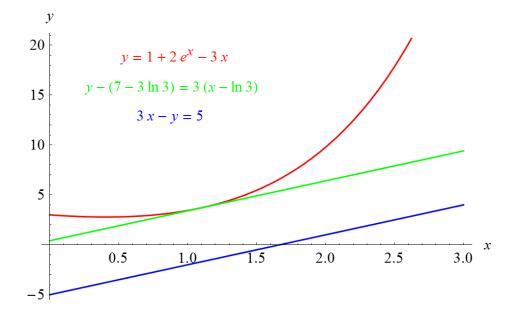
$$2e^{x} - 3 = 3$$
$$2e^{x} = 6$$
$$e^{x} = 3$$
$$x = \ln 3 \approx 1.099$$

Plug this value of x into the given function to get the corresponding y-value on the curve.

$$y(\ln 3) = 1 + 2e^{\ln 3} - 3(\ln 3) = 7 - 3\ln 3 \approx 3.704 \quad \Rightarrow \quad (\ln 3, 7 - 3\ln 3)$$

Finally, determine the equation of the line with slope 3 that goes through the point $(\ln 3, 7 - 3 \ln 3)$.

$$y - (7 - 3\ln 3) = 3(x - \ln 3)$$



Below is a plot of the given curve, the given line, and this tangent line parallel to the given line.