

Exercise 60

At what point on the curve $y = 1 + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 5$? Illustrate by graphing the curve and both lines.

Solution

Writing the given equation of the line as

$$y = 3x - 5,$$

we see that it has a slope of 3. The aim is to take the derivative of the given function and find where it's equal to 3.

$$\begin{aligned}y' &= \frac{d}{dx}(1 + 2e^x - 3x) \\&= \frac{d}{dx}(1) + \frac{d}{dx}(2e^x) - \frac{d}{dx}(3x) \\&= \frac{d}{dx}(1) + 2\frac{d}{dx}(e^x) - 3\frac{d}{dx}(x) \\&= (0) + 2(e^x) - 3(1) \\&= 2e^x - 3\end{aligned}$$

Set this equal to 3 and solve for x .

$$\begin{aligned}2e^x - 3 &= 3 \\2e^x &= 6 \\e^x &= 3 \\x &= \ln 3 \approx 1.099\end{aligned}$$

Plug this value of x into the given function to get the corresponding y -value on the curve.

$$y(\ln 3) = 1 + 2e^{\ln 3} - 3(\ln 3) = 7 - 3\ln 3 \approx 3.704 \quad \Rightarrow \quad (\ln 3, 7 - 3\ln 3)$$

Finally, determine the equation of the line with slope 3 that goes through the point $(\ln 3, 7 - 3\ln 3)$.

$$y - (7 - 3\ln 3) = 3(x - \ln 3)$$

Below is a plot of the given curve, the given line, and this tangent line parallel to the given line.

